THE PROBLEM OF THE LINEARIZATION OF THE EQUATIONS OF THE NONSTATIONARY NONISOTHERMAL FLOW OF A REAL GAS IN PIPELINES

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The analysis of numerical solutions is the basis for investigating the effect of various terms in the equations of the nonstationary nonisothermal motion of a real gas on the nature of the solution and its results in order to simplify and linearize the original equations to a form permitting analytical solution.

1. In the investigation of the nonstationary nonisothermal flows of a gas in pipes it is necessary to solve simultaneous equations in gas dynamics which have the following form for a cylindrical pipe of constant diameter [1]:

$$\frac{\partial\rho}{\partial t}+\frac{\partial(\rho w)}{\partial x}=0, \tag{1}$$

$$\frac{\partial(\rho w)}{\partial t}+\frac{\partial}{\partial x}(P+\rho w^2)=-\rho\left(gz'+\frac{\lambda}{2D}w|w|\right), \tag{2}$$

$$\frac{\partial}{\partial t}\left[\rho\left(g\frac{u}{A}+\frac{w^2}{2}\right)\right]+\frac{\partial}{\partial x}\left[\rho w\left(g\frac{u}{A}+\frac{P}{\rho}+\frac{w^2}{2}\right)\right]$$
$$=-\rho gz'w+\frac{1}{A}\left[\frac{4}{D}q(x,\ t)+\frac{\partial}{\partial x}\left(k_1\frac{\partial T}{\partial x}\right)\right], \tag{3}$$

$$P=\rho gz_0RT,\quad 0<x<L,\ t>0, \tag{4}$$

and the heat conduction equation for the external medium with appropriate initial and boundary conditions. To obtain the exact solution of the complete system of equations in finite analytical form encounters considerable mathematical difficulties in connection with which in the general case it is difficult to estimate the contribution of the separate terms in the above equations for arbitrary boundary conditions and, consequently, to justify deductions about simplifying the equations (1)-(4) and reducing them to a form permitting approximate analytical solution. Hence it is expedient to make estimates on the basis of the numerical solutions of problems in which the boundary conditions are specified as discontinuities in the boundary functions known to be greater than those encountered in actual problems.

Usually in solving problems in the nonstationary nonisothermal motion of a gas involving gas transfer in pipelines the following simplifications are possible: the flow of the gas is assumed to be one-dimensional; in the case of long pipelines and at gas velocities much less than that of sound, we can neglect terms in $w\partial w/\partial x$ and $\partial w/\partial t$ in the equations of motion and energy and also changes in the geometrical height z [1]. In addition, we can neglect heat transfer along the axis of the pipe because of the thermal conductivity of the gas; it is assumed that the temperature of the external medium is known and that the heat exchange law is taken in Newton's form [1].

To see that these assumptions are permissible it is necessary to compare the solution of the complete equations (1)-(4) with that of the equations simplified in accordance with the above conditions.

With the above assumptions, Eqs. (1)-(4) can be reduced after some algebra to the following form:

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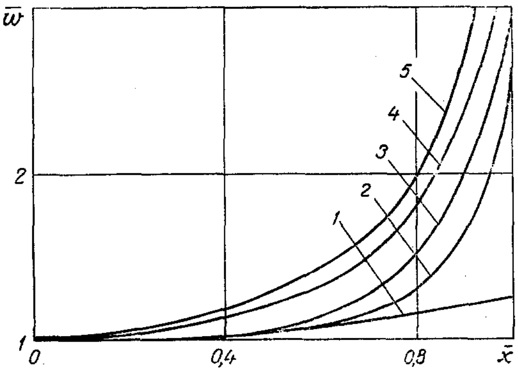


Fig. 1. Curves for the change in the nondimensional gas velocity along the length of the pipeline: 1) $\bar{t}=0$; 2) $\bar{t}=0.23$; 3) $\bar{t}=0.69$; 4) $\bar{t}=2.3$; 5) $\bar{t}=4.6$.

$$\frac{\partial G}{\partial x}=\frac{a}{z_0T}\left(\frac{P}{T}\cdot\frac{\partial T}{\partial t}+\frac{P}{z_0}\cdot\frac{\partial z_0}{\partial t}-\frac{\partial P}{\partial t}\right), \tag{5}$$

$$\frac{\partial P}{\partial x}=-b\,\frac{G^2z_0T}{P}, \tag{6}$$

$$\frac{\partial T}{\partial t}=-\frac{1}{a}\cdot\frac{Gz_0T}{P}\cdot\frac{\partial T}{\partial x}-bc\,\frac{G^3T^4z_0^2}{P^3}\left(\frac{\partial z_0}{\partial T}\right)_P+m\,\frac{T}{P}\left[z_0+T\left(\frac{\partial z_0}{\partial T}\right)_P\right]\frac{\partial P}{\partial t}+n\,\frac{(T^*-T)\,z_0T}{P}, \tag{7}$$

$$0<x<L,\quad t>0,$$

where

$$z_0=z_0(P,\ T);\quad c_p=c_p(P,\ T);\quad a=f/R;\quad b=\lambda R/2gDf^2;$$
$$c=AR^2/c_pf;\quad m=AR/c_p;\quad n=K\pi DR/c_pf.$$

In calculating the nonstationary nonisothermal gas flows described by Eqs. (5)-(7) we take the initial conditions as the distribution of the parameters of the steady flow along the pipe length at the initial moment of time ($t=0$). In the general case to determine the initial distribution we have to solve a Cauchy problem (with given conditions for the gas temperature and pressure at one end of the pipe), for the appropriate nonlinear system of ordinary differential equations to which (5)-(7) reduce for the steady motion of the gas.

We consider boundary conditions of the following form for Eqs. (5)-(7):

$$P(0,\ t)=f_1(t),\quad T(0,\ t)=f_2(t),\quad G(L,\ t)=\varphi(t),\quad t>0. \tag{8}$$

2. To solve Eqs. (5)-(7) numerically in $0\le x\le L$, $t\ge 0$, with the above initial distribution of the flow parameters and the boundary conditions (8), we use a finite difference method. Then (5)-(7) are first transformed to a system of quasi-linear equations of evolutionary type [2]. For this system we construct explicit second order finite difference schemes with step length h: the derivatives with respect to x are replaced at internal points by symmetric difference relations and at the boundary points by the appropriate one-sided three-point difference equations.* It is appropriate to use such schemes for systems of equations of quite complex form (5)-(7), while restrictions on the stability, as shown by investigations and practical calculations for the flows considered, do not lead to a significant reduction in the time step length τ.

To determine the pressure (P) and temperature (T) at points of the right boundary ($x=L$) in the case of condition (8), we must solve simultaneously a system of nonlinear differential equations at this boundary which follows from the above equations:

$$\frac{P}{T}\,\frac{1}{m\left(z_0+T\,\dfrac{\partial z_0}{\partial T}\right)}\cdot\frac{\partial T}{\partial t}-\frac{\partial P}{\partial t}=-\frac{\dfrac{1}{a}\,G\,\dfrac{\partial T}{\partial x}+bc\,\dfrac{G^3T^3z_0}{P^2}\cdot\dfrac{\partial z_0}{\partial T}-n\,(T^*-T)}{m\left(z_0+T\,\dfrac{\partial z_0}{\partial T}\right)\dfrac{1}{z_0}}, \tag{9}$$

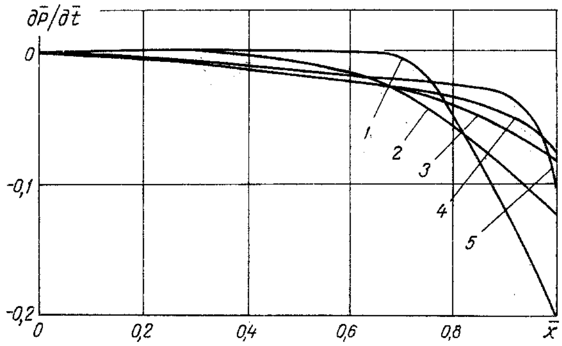
$$\frac{\partial P^2}{\partial x}=-2bG^2z_0T. \tag{10}$$

The finite difference approximation for Eqs. (9) and (10) at the boundary $\bar{x}=1$ with an accuracy of $O(h^2)$ leads to the following nonlinear equations for the pressure and temperature:

$$\bar{P}^2_{n,k+1}=\frac{1}{3}\left(4\bar{P}^2_{n-1,k+1}-\bar{P}^2_{n-2,k+1}\right)-\frac{2}{3}\,hA_1\bar{\varphi}^2z_0\left(\bar{P}_{n,k+1},\ \bar{T}_{n,k+1}\right)\bar{T}_{n,k+1}, \tag{11}$$

$$\bar{T}_{n,k+1}=\frac{\bar{P}_{n,k+1}-C}{B}\quad (k=0,\ 1,\ 2,\ \ldots), \tag{12}$$

*The transformed system of Eqs. (5)-(7) was solved numerically on a computer in nondimensional form. The nondimensional variables were: $\bar{x}=x/L$; $\bar{t}=t/t_0$; $\bar{P}=P/P_c$; $\bar{G}=G/G_0$, where $t_0=L/c_0$; P_c, T_c were the critical pressure and temperature; c_0 is the speed of sound in the gas.



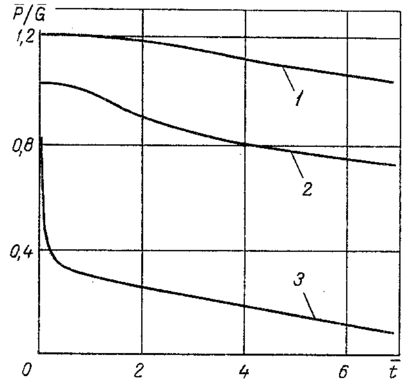


Fig. 2 Fig. 3

Fig. 2. Change in the pressure derivative with time along the length of the pipeline: 1) $\bar{t} = 0.25$; 2) $\bar{t} = 0.75$; 3) $\bar{t} = 2.25$; 4) $\bar{t} = 4.25$; 5) $\bar{t} = 6.50$.

Fig. 3. Change in $\bar{P}/\bar{G}$ with time: 1) $\bar{x} = 0$; 2) $\bar{x} = 0.5$; 3) $\bar{x} = 1.0$.

where

$$A_1 = \frac{2bLT_c G_0^2}{P_c^2}, \quad B = \frac{1}{m} \cdot \frac{\bar{P}_{n,k}}{\bar{T}_{n,k}} \cdot \frac{1}{\left(z_0 + \bar{T}\frac{\partial z_0}{\partial \bar{T}}\right)_{n,k}};$$

$$C = \frac{\tau t_0}{mP_c} \cdot \left[\frac{1}{a} G_0 \bar{G}_{n,k} \frac{T_c}{L} (3\bar{T}_{n,k} + \bar{T}_{n-2,k} - 4\bar{T}_{n-1,k}) + bcG_0^3 \bar{G}_{n,k}^3 \left(\frac{\bar{T}^3}{\bar{P}^2} z_0 \frac{\partial z_0}{\partial \bar{T}}\right)_{n,k} - nT_c(\bar{T}^* - \bar{T}_{n,k})\right]$$
$$\times \left[\left(z_0 + \bar{T}\frac{\partial z_0}{\partial \bar{T}}\right)_{n,k} \frac{1}{(z_0)_{n,k}}\right]^{-1} + \bar{P}_{n,k}\left[1 - \frac{1}{m} \cdot \frac{1}{\left(z_0 - \bar{T}\frac{\partial z_0}{\partial \bar{T}}\right)_{n,k}}\right].$$

To determine the values of $\bar{P}_{n,k+1}$ from (11) we use an iteration method:

$$\bar{P}_{n,k+1}^{(n+1)} = \left[\frac{1}{3}(4\bar{P}_{n-1,k+1}^2 - \bar{P}_{n-2,k+1}^2) - \frac{2}{3} hA_1 \bar{\varphi}^2 z_0 \left(\bar{P}_{n,k+1}^{(n)}, \frac{\bar{P}_{n,k+1}^{(n)} - C}{B}\right) \frac{\bar{P}_{n,k+1}^{(n)} - C}{B}\right]^{1/2}. \tag{13}$$

We take $\bar{P}_{n,k+1}^{(0)} = (1/3)(4\bar{P}_{n-1,k+1}^2 - \bar{P}_{n,k+1}^2)$ as the zero order approximation in the iteration process (13).

For an ideal gas ($z_0 = 1$), Eq. (11) can be solved for $\bar{P}_{n,k+1}$ in finite form:

$$\bar{P}_{n,k+1} = -\frac{1}{3} A_1 h \bar{\varphi}^2 \frac{1}{B} + \sqrt{\left(\frac{1}{3} A_1 h \bar{\varphi}^2 \frac{1}{B}\right)^2 - \frac{1}{3}\left(\bar{P}_{n-2,k+1}^2 - 4\bar{P}_{n-1,k+1}^2 - 2A_1 h \frac{C}{B} \bar{\varphi}^2\right)}. \tag{14}$$

3. The above finite difference schemes and algorithms were used to calculate the nonstationary nonisothermal flows of a real gas in pipelines for the following purposes: to estimate the effect of the inertial terms $\partial(\rho w)/\partial t$ and $\partial(\rho w^2)/\partial x$ in the equations of the nonstationary nonisothermal motion of a gas in pipelines (1)-(4); to investigate the effect of the actual properties of the gas on the nature of the changes in the parameters of the gas flow by comparison with the case of an ideal gas; to investigate the nature of the changes in the various terms in the original equations so as to simplify them and obtain linearized equations permitting approximate analytical solutions.

a) From the results of the calculations we compared the numerical solutions of Eqs. (1)-(4) under the above conditions with and without the terms $\partial(\rho w)/\partial t$ and $\partial(\rho w^2)/\partial x$ for the following boundary conditions ($t > 0$):

$$P(0, t) = f_1(t), \quad T(0, t) = f_2(t); \quad \rho w(L, t) = 0. \tag{15}$$

TABLE 1. Comparison of the Distributions of the Parameters of Steady Nonisothermal Flow With and Without Changes in the Velocity Head (Actual gas)

x, km	P, atm		T, °C	
	I	II	I	II
0	55,0	55,0	47,0	47,0
10	53,25	53,22	42,0	42,0
20	51,44	51,41	37,5	37,5
30	49,60	49,56	33,5	33,4
40	47,70	47,66	29,9	29,8
50	45,78	45,72	26,6	26,5
60	43,79	43,70	23,7	23,6
70	41,71	41,61	21,0	21,0
80	39,54	39,43	18,6	18,5
90	37,26	37,13	16,4	16,3
100	34,85	34,70	14,4	14,3

Note. I corresponds to the solution taking account of changes in the velocity head; II to the solution ignoring changes in the velocity head. The initial conditions were: $P(0) = 55$ kG/cm^2; $T(0) = 47°C$; $\rho w(x) = 25$ kg · sec/m^3; $0 \leq x \leq 100$.

At the initial moment of time ($t = 0$) the gas flow was steady. The case under consideration is limiting and this makes it possible to estimate the maximal possible divergences. Physically this problem corresponds to the case of the instantaneous covering of the right end of the pipe.

The solution of Eqs. (5)-(7), ignoring the inertial term, was obtained using the above algorithm; in the example considered it was compared with the numerical results obtained for the initial and boundary conditions given below and the same initial data for the system of equations which takes account of the inertial terms [3]. The conditions for the numerical example, the computational results, and the corresponding estimates are given in [2].

We also compared the solutions with and without the inertial terms for stationary gas flow (Table 1). It follows from the analysis of the results that the inertial terms have insignificant effect on the gas flow conditions in main pipelines.

b) The calculations made is possible to analyze for the flows considered the limits of the deviations of the gas flow parameters and the errors occurring as a result of ignoring the actual properties of the gas in calculating the pressure and temperature distributions of the gas along the pipeline [2].

In addition, to estimate the effect of the gas being thermodynamically not ideal on the nature of the flow conditions we considered the case of a discontinuous increase in the gas flow rate at the end of the pipe. This case is limiting in the sense that in real conditions, with the exception of the case where there are discontinuities in the pipelines, all possible changes in ρw are smaller in magnitude and more drawn out in time.

The conditions for the numerical example in the second case were as follows. The initial conditions for the calculation of the steady flow were $P(0,0) = 55 \cdot 10^4$ kG/m^2; $T(0,0) = 320°K$; $G(0,0) = 100$ kg/sec. The boundary conditions ($t > 0$) were: $P(0,t) = 55 \cdot 10^4$ kG/m^2; $T(0,t) = 320°K$; $G(L,t) = 200$ kg/sec.

The calculations were made with the following values of the parameters: $L = 100$ km; $D = 0.7$ m; $P_c = 45.8 \cdot 10^4$ kG/m^2; $T_c = 190.6°K$; $\lambda = 0.012$; $R = 50$ kGm/kg · °C; $c_p = 0.5$ kcal/kg · °C, $T^* = 275°K$; $K = 2.0$ kcal/m^2 · h · °C.

Bertleau's form of the state equation was used as the equation of state for the real gas.

Analysis of these numerical calculations in particular showed that the curves for the depression of the temperature along the length of the pipelines for an actual gas are lower than the corresponding curves for an ideal gas because the gas is thermodynamically not ideal. As x increases (i.e., the depression of the pressure) the temperature difference between a real and an ideal gas increases. At the end of the pipe, for example, when $\bar{t} = 4.6$ (20 min) this difference reaches a very significant value, 0.06 (~11.4°C) for the case under consideration; for $\bar{t} = 0$ the discrepancy between the temperatures is 0.017 (~3.3°C) at the end of the pipe. The calculations show that the pressure of a real gas in time remains higher than that of an ideal gas. For $\bar{t} = 0$ the discrepancy is 0.04 (~$1.8 \cdot 10^4$ kG/m^2) at the end of the pipe. As the time increases the difference between the pressure of a real gas and that of an ideal one increases and in the given example reaches $\Delta\bar{P} = 0.17$ ($7.6 \cdot 10^4$ kG/m^2) for $\bar{t} = 4.6$. At points far from the right boundary the difference is 0.001 -0.002 ($0.045-0.9 \cdot 10^4$ kG/m^2).

Consequently, that the gas is thermodynamically not ideal has a marked effect on the characteristics of the nonstationary gas flow when there are sharp (discontinuous) changes in the boundary conditions. In these cases we have to make calculations for pipelines taking these changes into account, i.e., starting from the equations for a real gas. Since there are no experimental data for nonstationary nonisothermal gas flow conditions, the results of the numerical calculations for stationary gas flow were compared with Shorre's experimental data [4]. The temperature distribution obtained on a computer was very close (within 1-1.5%) to Shorre's data.

c) The numerical solutions of the equations for the nonstationary nonisothermal motion of a real gas were used to study the nature of the changes in the various terms and combinations of terms in (5)-(7) in order to clarify the possibility of linearizing them to obtain approximate analytical solutions. The virtually exact numerical solutions of the complete system of equations can be used as reference solutions for estimating the accuracy of the approximate analytical solutions of the linearized equations.

Analysis of the structure of Eqs. (5)-(7) shows that the basic nonlinearity is due to terms containing the derivatives $(\partial z_0/\partial T)_p$, $\partial z_0/\partial t$, $\partial P/\partial t$, and also terms proportional to G^2.

Hence it is primarily of interest to estimate the limits and nature of the changes in these terms in the equations from the results of the numerical calculation.

As the calculations showed, the compressibility coefficient varies within comparatively narrow limits even when the flow rate (cf. para. b) changes discontinuously, i.e., we can take $z_0 \cong (z_0)_{av}$.

Similar conclusions can be made about the behavior of the function $(\partial z_0/\partial T)_p$, which makes it possible to average it. This is equivalent to the practical possibility of writing the differential equations for the enthalpy in the form [5]:

$$di = c_p dT - (c_p D_i)\, dP.$$

Then we can analyze the various methods of averaging the term determining the friction loss.

Charnyi [1] proposed methods for linearizing the equations of nonstationary nonisothermal motion of a gas. One of the basic methods which he proposed assumes that it is possible to average the term in the equations of motion determining the square of the friction, which makes it possible to reduce the equation for the nonstationary nonisothermal motion to the heat conduction equation (if the inertial forces are ignored) or to the wave equation (if friction forces along the pipe length are ignored).

Fig. 1 shows the gas velocity along the pipeline length as a function of the time.* It follows from Fig. 1 that the gas velocity varies significantly in the zone where perturbations (gas takeoff) occur. Here $\bar{w}$ increases by a factor 3-4 depending on the time, by comparison with its average value.

On the basis of the results we can conclude that this method of averaging can only be used to calculate the pressures at points sufficiently far from the location of the perturbation and when the durations of the transient processes are large. In the remaining cases the method can lead to considerable errors.

Then we can discuss the method of linearization based on averaging the pressure derivative with respect to the time, as used in [6-9]. Fig. 2 shows $\partial\bar{P}/\partial\bar{t}$ along the length of the gas pipe at various moments of time for the conditions of the problem in [10]. We see that near the boundary, where a perturbation occurs, $\partial\bar{P}/\partial\bar{t}$ varies significantly with time and along the length of the pipeline and it cannot be averaged. If it is of practical interest to solve the problem for $\bar{x} < 0.6$, it is not permissible to average the derivative.

Thus, for problems with rapid oscillations of the boundary conditions (start-up conditions, pipeline shutdown, rapid increase in gas takeoff), this method of linearizing is insufficiently exact.

In [11] and other papers Leibenzon's proposal to substitute $\tau = \frac{D}{\lambda}\int_0^t \frac{P}{G}\,dt$ $\left(\text{or}\quad \tau \cong \frac{D}{\lambda}\left(\frac{P}{G}\right)_{av} t\right)$ was used and this makes it possible to reduce the equations of the nonstationary nonisothermal motion of a gas to the heat conduction equation in the square of the pressure.

Fig. 3 shows $\bar{P}/\bar{G}$ as a function of the time at various points of the pipeline ($x = 0$, 0.5, and 1.0) for the conditions of the problem given in [13]. We see that $\bar{P}/\bar{G}$ varies markedly with time and also more rapidly along the length of the pipeline. Thus, this method of linearizing when there are rapid oscillations of the gas flow rate can lead to significant errors in the computations.

In addition, from the results of the numerical calculations we estimated the order of magnitude of the remaining derivatives ($\partial\bar{P}/\partial\bar{x}$, $\partial\bar{P}/\partial\bar{t}$, $\partial\bar{T}/d\bar{x}$, $\partial\bar{T}/\partial\bar{t}$). Numerical analysis showed that these derivatives have the following orders of magnitude:

*The conditions for the example are given in b).

$$\frac{\partial \bar{P}}{\partial \bar{t}} \sim (10^{-1} - 10^{-2}), \quad \frac{\partial \bar{P}}{\partial \bar{x}} \sim 1,$$

$$\frac{\partial \bar{T}}{\partial \bar{t}} \sim (10^{-2} - 10^{-3}), \quad \frac{\partial \bar{T}}{\partial \bar{x}} \sim 1.$$

Noting the nature of the change in these derivatives, we can reduce the Eqs. (5)-(7) to a simpler form:

$$\frac{\partial G}{\partial x} = -\frac{a}{z_0 T} \cdot \frac{\partial P}{\partial t},$$

$$\frac{\partial P}{\partial x} = -b\,\frac{G^2 z_0 T}{P},$$

$$\frac{\partial T}{\partial x} = -abc\,\frac{G^2 T^3 z_0}{P^2}\left(\frac{\partial z_0}{\partial T}\right)_P + \frac{na}{G}(T^* - T).$$

For small changes in the temperature along the pipeline ($\partial T/\partial x \to 0$, $T \cong T^*$) Eqs. (16) become the familiar equations for nonstationary isothermal motion in Charnyi's form [1]. The comparison of the numerical solutions of Eqs. (5)-(7) with the approximate solutions based on the linearized equations (16) for the case of an ideal gas [12] showed that they coincided sufficiently closely to be quite acceptable for practical applications.

NOTATION

P, ρ, w	are the mean pressure, density and gas velocity at a cross section of the pipe;
u	is the internal energy;
A	is the thermal equivalent of mechanical work;
λ	is the hydraulic drag coefficient;
f, D	are the cross sectional area and pipeline diameter;
g,	is the acceleration due to gravity;
$z(x)$	is the ordinate of pipe axis measured from the horizontal plane;
z_0	is the compressibility coefficient;
R	is the gas constant;
T, T^*	are the temperature of gas and external medium;
q	is the thermal flux across pipe wall per unit time and per unit wall area;
k_1	is the gas heat conduction coefficient;
c_p	is the isobaric gas heat capacity;
x	is the coordinate along pipeline axis;
t	is the time;
τ	is the time step length;
h	is the coordinate step length.

LITERATURE CITED

1. I. A. Charnyi, The Fundamentals of Gas Dynamics [in Russian], Gostoptekhizdat (1961).
2. V. P. Radchenko and B. L. Krivoshein, Inzh.-Fiz. Zh., 19, No. 4 (1970).
3. A. F. Voevodin, Izv. Sibirsk. Otdeleniya AN SSSR, Ser. Tekh. Nauk, No. 8, Issue 2 (1969).
4. Ch. Shorre, Oil and Gas J., 53, No. 21 (1954).
5. N. I. Belokon', Thermodynamics [in Russian], Gosénergoizdat (1954).
6. Yu. D. Sokolov, Ukrainsk. Matem. Zh., 5, No. 2 (1953).
7. F. G. Tempel', Author's abstract of candidate's dissertation [in Russian], VNII, Moscow (1958).
8. A. Sh. Asutaryan, Z. T. Galiullin, and V. I. Chernikin, Izv. Vuzov. Neft' i Gaz, No. 10 (1961).
9. I. E. Khodanovich, Analytical Foundations of the Design and Operation of Main Pipelines [in Russian], Gostoptekhizdat (1961).
10. B. L. Krivoshein, E. M. Minskii, V. P. Radchenko, I. E. Khodanovich, and M. G. Khublaryan, Inzh.-Fiz. Zh., 13, No. 4 (1967).
11. Z. T. Galiullin, "Nonstationary gas motions in main pipelines", Trudy VNIIgaza, Nos. 29/37, Nedra, Moscow (1966).
12. I. M. Konstantinova, Yu. P. Korotaev, and B. L. Krivoshein, "Problems in natural gas transport", Trudy VNIIgaza, Nos. 38/46, Nedra, Moscow (1970).
13. B. L. Krivoshein, V. P. Radchenko, and M. N. Abdullaev, Za Tekhnicheskii Progress, No. 5 (101), Baku (1969).